Riemann Sum Notation

1. Explain why $\sum_{k=1}^{N} f\left(3 + \frac{(k-1)}{N}\right) \cdot \frac{1}{N}$ is an under approximation of the area bounded by the graph of f and the x-axis when f is increasing on the interval $[3, 4]$.

2. The function $y = g(t)$ represents the relationship between the rate of change in the value of investment stocks (in dollars per month) and the number of months t elapsed since the stocks were purchased. Which of the following sums approximates the change in the value of the stocks over the interval of time from 4 to 7 months after the stocks were purchased?

   (a) $\sum_{k=4}^{7} g(k)$

   (b) $\sum_{k=4}^{7} g(t) \cdot \Delta t$

   (c) $\sum_{k=1}^{6} g(4 + .5k) \cdot .5$

   (d) $\sum_{k=0}^{3} g(4 + k) \cdot \Delta t$

   (e) $\sum_{k=0}^{3} g(4 + k)$

3. Let $f(x)$ represent the linear density (in g/m) of a 20 meter long wire, where x is the distance in meters from one end. The mass of the wire is approximated by the left endpoint approximation with N terms:

$$\sum_{i=1}^{N} f((i-1)\Delta x) \cdot \Delta x$$

   Note that this Riemann sum is based on a uniform partition.

   Explain what the following expressions represent **in the context of the wire** and provide its units of measurement.

   (a) Δx

   (b) $(i-1)\Delta x$

   (c) $f((i-1)\Delta x)$

   (d) $f((i-1)\Delta x) \cdot \Delta x$

   (e) $\sum_{k=1}^{N} f((i-1)\Delta t) \cdot \Delta t$

4. Let $f(t)$ represent the horizontal velocity (in ft/s) of a golf ball t seconds after it was struck and lands b seconds later. The horizontal distance traveled by the golf ball is approximated by the right endpoint approximation with Nterms:

$$\sum_{i=1}^{N} f(i\Delta t) \cdot \Delta t$$

Note that this Riemann sum is based on a uniform partition.

Explain what the following expressions represent **in the context of the golf ball** and provide its units of measurement.

(a) Δt

(b) $i\Delta t$

(c) $f(i\Delta t)$

(d) $f(i\Delta t) \cdot \Delta t$

(e) $\sum_{k=1}^{N} f(i\Delta t) \cdot \Delta t$

5. The function $g(t)$ gives the rate at which oil leaves a tanker, and is decreasing between 2 minutes and 10 minutes. Which of the following are underestimates of the amount of oil that left the tank between 2 and 10 minutes.

(a) $g(2) + g(3) + g(4) + g(5) + g(6) + g(7) + g(8) + g(9)$

(b) $\sum_{k=3}^{10} g(k)$

(c) $\sum_{j=1}^{20} g\left(2 + \frac{j-1}{2}\right) \cdot \frac{1}{2}$

(d) $\sum_{j=1}^{20} g\left(2 + \frac{j}{2}\right) \cdot \frac{1}{2}$

(e) $2 \cdot (g(4) + g(6) + g(8) + g(10))$

(f) $2 \cdot (g(3) + g(5) + g(7) + g(9))$